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The Study of Grey Relational Analysis and Artificial Neural Network for Forecasting Carbon Price

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A B S T R A C T

This paper uses economic variables and financial market indices, such as Baltic Dry Index, PJM electricity price, Brent crude oil future price, steel price index, Hot-Rolled Coil Steel Futures, interest rate, unemployment rate, stock index, money supply (M2), consumer price index, industrial production index, and Commodity Research Bureau futures index, which are related to the carbon price. Gray relational analysis (GRA) is employed in this research to determine variable rankings, and artificial neural network (ANN) models are utilized to forecast carbon prices. According to the results, the predictions of the high gray relation variables were superior to those of the low gray relation variables. Four ANN models, namely, feedforward with backpropagation network (BPN), principal component analysis network (PCA), radial basis function (RBF), and recurrent neural networks (RNN), were compared. The GRA model helped simplify the ANN model to facilitate carbon price forecasting, and the ANN model can effectively reduce forecasting errors. The results indicate that the RNN is more suitable for forecasting EU carbon allowances (EUA).

Keywords: Carbon Futures, Grey Relational Analysis (GRA), Artificial Neural Network (ANN)

JEL Classification: G13, G17

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1. Introduction

Many countries have created carbon trading markets for the past few years to help reduce carbon emissions and avoid extreme climate change effects if global temperatures rise about 1.5 degrees Celsius relative to pre-industrial levels. The Kyoto Protocol in 1997 includes three mechanisms to achieve its goals for reducing greenhouse gas (GHG) emissions, including the Emissions Trading Scheme (ETS), The Clean Development Mechanism (CDM), and Joint Implementation (JI). These mechanisms have produced certified emission reductions and emission reduction units that enable both quotas and credits to be traded in the marketplace. ETS are trading systems, often called cap-and-trade systems, whereby a country, region, or regulatory body caps the total level of GHG emissions to help ensure that required emission reductions will occur to keep emissions (in aggregate) within a prescribed carbon budget. Companies in industries with low emissions can sell their extra allowances to larger emitters who buy these carbon credits to compensate for their greenhouse emissions, with supply and demand in ETS markets setting the price for greenhouse gas emissions, known as the “carbon price.”

In spite of the increasing volume of economic transactions linked to the carbon market, the carbon market is becoming more and more important. To achieve national emission reduction targets and comprehend the dynamics of the carbon trading market, it is necessary to have an accurate forecast of carbon prices. Carbon prices are affected by a number of factors, making forecasting difficult. Price forecasting has been heavily influenced by deep learning models in recent years due to their high forecasting accuracy when dealing with nonlinear time series data. This study seeks to understand the non-linear dynamics of carbon prices through a two-way process that enhances forecast accuracy.

The first step involves using gray relational analysis (GRA) to determine which factors impact carbon prices most by their relevant ranks. These factors include Baltic Dry Index (BDI), PJM Electricity price (PJM), Brent crude oil price (Futures), Dow Jones Iron & Steel (DJUSST), the US Midwest Domestic Hot-Rolled Coil Steel Futures (HRCc1) for finance market indices. Meanwhile, economic variable are the US unemployment rate, money supply (M2), consumer price index (CPI), industrial production index (IPI), the10-Year Bond Yield, Dow Jones Industrial Average (DJI), and the Commodity Research Bureau (CRB) futures price index.

Previous literature proved the efficiency of GRA in selecting the best alternatives among possible multiple choices of key factors (Feng and Wang, 2000; Kung and Wen, 2007; Hamzaçebi and Pekkaya, 2011).

The second procedure is the utilization of four Artificial Neural Network (ANN) models, namely, backpropagation perceptron network (BPN), principal component analysis network (PCA), recurrent neural network (RNN), and radial basis function neural network (RBFNN), to examine the best training data sets for carbon prices. According to Huang et al. (2008), Zhang and Xiao (2000), and Kim (1998), ANN models have satisfactory prediction accuracy.

The present study has the following objectives:

(1) To identify the strongest determinants that affect carbon prices by ranking their gray relational grades (GRGs);

(2) To test four ANN models, which are powerful forecasting tools for predicting carbon pricing;

(3) To determine which of the four ANN models has the highest capacity for forecasting based on the mean absolute error (MAE) and root mean square error (RMSE) tests.

(4) To combine the initial findings of the GRA and ANN models and separate the determinants (GRGs) among those with high and low GRGs.

The group of determinants (all variables, high-GRG variable, and low-GRG variable) is measured by ANN models. This process aims to confirm whether the GRA findings are consistent with those of the ANN results by uncovering the factors that affect carbon price performance.

The importance of influencing factors in carbon price prediction has been overlooked by previous studies, who based their forecast on historical time series of carbon prices. Various factors affect the fluctuation of carbon prices in the world's carbon emissions trading market, which is relatively complex. All the information cannot be summarized by the historical data from the time series of carbon prices. This paper helps to consider economic factors and financial market indices as important determinants, thereby enhancing the authenticity and accuracy of forecasts. Expand existing research on carbon pricing forecasting and explore the efficiency and feasibility of deep learning in carbon pricing forecasting. In this study, ANN is utilized to analyze the prediction results of the GRA model. Further, this study utilized the BPN, PCAs, RBF, and RNN models of neural networks to introduce carbon price forecasting. The ANN model is superior in carbon price prediction.

The paper is organized as follows: Section 2 presents the literature review. Section 3 explains the GRA and the four ANN models. Section 4 provides a summary of the data and empirical test results, with emphasis on data and empirical results. Section 5 provides the conclusions.

2. Literature Review

Global warming, caused by greenhouse gases, is the biggest threat to human life if not addressed properly. Some initiatives that address global warming were the United Nations Framework Convention on Climate Change in 1992, the Kyoto Protocol in 1997, and the Bonn Agreement in 2001. Global warming, caused by greenhouse gases, is the greatest threat to human life without proper treatment.

The carbon trading market is an efficient way to reduce carbon emissions. Accurate carbon price projections are crucial for policymakers and investors. However, current forecasting models do not accurately predict carbon prices because of the non-linearity, uncertainty, and complexity of carbon prices. Current price studies primarily assess the relationship between carbon prices and energy prices. Mansanet et al. (2007) identified carbon prices and extreme weather events as drivers of energy market prices (i.e., oil, gas, and coal). Alberola et al. (2008) established a model of carbon and energy prices and examined the effects of changes on carbon prices. The impacts of structural breakpoints from 2005 to 2007 in the European Union (EU), ETS was also studied. The results showed that energy prices and unanticipated temperature changes influenced carbon prices. Liu et al. (2007) emphasized the relation between carbon and energy prices (such as energy, oil, coal and natural gas). They found that carbon dioxide emission was restricted to the impact of changes in electricity and heat emission rates. Oberndorfer (2009) examined the factors influencing power companies' stock prices in the EU ETS and determined that electric power firms must have a dynamic carbon price to maintain profitability. The results show that the carbon price is positively correlated with power, even though this effect is not asymmetric, and that the carbon market effect varies by time and country.

Other studies use different modeling methods for forecasting carbon prices. Benz and Truck (2009) used a Markov-switching model to analyze the carbon price and applied the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to calculate the return volatility. Chevallier (2011a) applied Markov-switching Vector AutoregRession (VAR) model to evaluate the business cycle effect for carbon price and exercised threshold vector error correction to obtain the interrelationship between the macroeconomic variables and carbon price. Zhu et al. (2013) propose using an Autoregressive Integrated Moving Average (ARIMA) and Least Squares Support Vector Machines (LSSVM) for the predicating of carbon prices. The empirical results demonstrated that the proposed mingle methodology was suitable for carbon price forecasting.

Some articles have examined the economic and financial variables impacting carbon prices. Oberndorfer (2009) examined the relationship between EUA Allowance price fluctuation and the return on electricity equity and found a positive correlation between EUA price changes and stock returns. The difficulty in identifying volatility spillovers was pronounced for both markets. Frunza et al. (2010) forecasted the return of Carbon Allowance and considered factors such as oil, gas, coal, and equity index to explain the evolution of carbon prices. Chevallier (2011b) used Factor-Augmented Vector Autoregression) FAVAR model and regime switching model to evaluate the linkage of international shocks to the carbon market. Their relative variables include macroeconomic, financial, and commodity variables. The study revealed that macroeconomic variables had a lag in their impact on carbon prices, and the regime-switching model was able to capture industrial production and carbon prices.

Due to their improved prediction accuracy and reliability, deep learning models have become the most popular research method in recent years. In a certain way, these addresses the shortcomings of conventional machine learning. Traditional machine learning and deep learning were employed by Wang et al. (2017) to predict four different data sets. Deep learning demonstrated a superior ability to predict. The introduction of a neural network by Movagharnejad et al. (2011) enabled them to forecast the price of commercial oil in these crude oils. Deep recurrent neural network (RNN) models for long-term electric load prediction were developed and optimized by Rahman et al. (2018).

Zhang and Wen (2022) used an advanced deep neural network model to forecast carbon prices, showing superior prediction performance that achieves forecasting accuracy with the lowest Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) values. Furthermore, the Recurrent Neural Network (RNN) model in deep learning is a highly effective way to adjust time series data. For example, Nelson et al. (2017) built the Long Term Memory (LTM) model to forecast the future trend in share prices. The LSTM model's effectiveness in stock price prediction was confirmed through historical time series data. The model used by Cen and Wang (2019) for predicting the prices of West Texas Intermediate crude oil and Brent crude oil was LSTM, and it achieved a higher prediction accuracy.

3. The Data and Methodology

3.1 Data

This study aims to predict carbon prices from three distinct datasets collected from the European Union Allowances (EUA) futures, IHS Markit Global Carbon Index (GLCARB), and The KraneShares Global Carbon Strategy ETF (KRBN) monthly settlement price. The period of data is from January 2009 to March 2023, and the monthly data are summarized in Table 1. EUA is the most liquid market. The GLCARB is designed to measure the performance of the global carbon credit market, to measure its performance. European Union Allowances (EUA), California Carbon Allowances (CCA), and Regional Greenhouse Gas Initiative (RGGI) are the leading European and North American cap-and-trade programs currently covered by the index. However, the GLCARB was the first Carbon Index to combines proprietary information and futures markets data in 2014. KraneShares Global Carbon Strategy ETF (KRBN) is currently the most significant carbon ETF in the market.

Table 1. Carbon Price Data

Variables	Data period	Title	**Source**
EUA	2009.01-2023.03	European Union Allowance (EUA)	https://www.investing.com
GLCARB	2014.08-2023.03	IHS Markit Global Carbon Index （GLCARB）	S&P Dow Jones Indices website
KRBN	2020.08-2023.03	KraneShares Global Carbon Strategy ETF（KRBN）	https://www.investing.com

Economic variables have been extracted to analyze the impact on carbon price (Kanen, 2006; Fæhn et al., 2009; Oberndorfer, 2009; Hintermann, 2010; Chevallier, 2011a, 2011b, 2011c, 2009d). Previous research indicated the effects of interest rate, stock index, M2, IPI, CPI, and oil price on relative carbon price (Tucker, 1995; Chevallier, 2009; Oberndorfer, 2009; Zagaglia, 2010; Niu et al., 2011). They showed that economic variables react to carbon pricing. Fæhn et al. (2009) found that the carbon market allows combining with revenue recycling by wages tax reductions to reduce the unemployment rate and increase employment. Numerous studies exposed financial and commodity indicators to be related to carbon prices or oil prices, such as the CRU steel price index, CRB futures index, and Spark Spread (Demailly and Quirion, 2008; Chevallier, 2009d; Frunza et al., 2010; Lin and Sim, 2013; Thema et al., 2013). Lin and Sim (2013) considered that international freight was the "engine of growth" by which the goal of economic development can be achieved, with the Baltic Dry Index (BDI) as the proxy variable.

The input variables include the unemployment rate, interest rate, consumer price index, industrial production index, money supply, stock price index, Iron & Steel index, Hot-Rolled Coil Steel price, Brent crude oil price, BDI, CRB index, and PJM electricity price in this study, as shown in Table 2.

Table 2. The Data of Financial Variable

Variables	**Code**	**Source**
Unemployment rate	X1	https://www.bls.gov
United States 10-Year Bond Yield	X2	https://www.investing.com
Consumer price index (CPI)	X3	
Industrial production index (IPI)	X4	
Money supply (M2)	X5	https://research.stlouisfed.org
Dow Jones Industrial Average (DJI)	X6	https://www.investing.com
Dow Jones Iron & Steel (DJUSST)	X7	
US Midwest Domestic Hot-Rolled Coil Steel Futures (HRCc1)	X8	
Brent crude oil price (Futures)	X9	
Baltic Dry Index (BADI)	X10	
CRB Commodity Index	X11	http://www.cnyes.com
PJM Electricity price (PJM)	X12	https://www.monitoringanalytics.com

3.2 Methodology

1. Grey Relational Analysis (GRA)

The grey system theory, which was introduced by Deng (1982), is a grey system containing partially known and unknown data. GRA is a part of gray system theory, which is suitable for answering questions with complex interrelationships among multiple factors.

Deng (1989) states that the methodology has a degree of discrepancy between two data sequences based on Grey Relational Grade. GRA is a quantitative approach for evaluating some variables' relationships, while $x_0(k)$ is reference sequences and $x_i(k)$ is comparative sequences in terms of the gray relational coefficient $\psi\left(x_0^{(0)}, x_i^{(0)}\right)$. It can be shown as follows:

$$\psi\big(x_0(k), x_1(k)\big) = \frac{\Delta_{min} + \xi\Delta_{min}}{\Delta_{0i}(k) + \xi\Delta_{max}}$$
$$= \frac{min_{\forall i}^{\cdot} \cdot min_{\forall k}^{\cdot}|x_0(k) - x_i(k)| + Zmax_{\forall i} \cdot max_{\forall k} \cdot |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + Zmax_{\forall i} \cdot max_{\forall k} \cdot |x_0(k) - x_i(k)|}, \tag{1}$$

where $Z \in (0,1)$ stand for the distinguishing coefficient and is usually set to 0.5. After calculating the whole gray relational coefficient, the GRG can then be measured as follows:

$$\psi(x_0, x_i) = \frac{1}{n}\sum_{k=1}^{n} \psi\big(x_0(k), x_i(k)\big). \tag{2}$$

When the gray relational level of the series is derived, the reference data sequence xi can be ranked based on their relative degrees.

2. Artificial Neural Network (ANN)

ANN is an information operating system that simulates the human brain's functionality as it relates to human thought and learning. The neural network comprises neurons, connections, and the learning algorithm.

This paper used four neural networks (Chang and Chang, 2005) as follows:

(1) Back-propagation Network (BPN)

ANN models are a common choice for the BPN model, which is one of the most popular types. Back Propagation was utilized by Charkha (2008) to predict trends, but Radial Basis Network was more effective in predicting stock prices. Ma et al. (2010) found that the BP neural network could be effective in predicting the short-term trend of the stock market. The BP algorithm theory is based on the error correction learning rule, which utilizes the error function to change connection weights and gradually reduce errors. To simplify, this error is the difference between the actual network output and the desired output. Its architecture has multilayer perceptions that utilize backpropagation error as a learning algorithm for multilayer perceptions. There are three layers in the feedforward network: input, hidden, and output. Note that the first layer is input, the second layer contains the individual neurons of the input layer, and the third is the output values of these neurons, which are forwarded to the neurons in the hidden layer. Equation (3) indicates the expression of the error on the *jth* layer. This algorithm's derivations are based on the error function illustrated in Equation (4) for normalization purposes.

$$e_j = \big(t_j + o_j\big)\ , \tag{3}$$

$$E = \tfrac{1}{2}\textstyle\sum_j e_j^2 = \tfrac{1}{2}\sum_j\big(t_j + o_j\big)^2\ , \tag{4}$$

where e_j stand for the layer index, t_j refer to the desired output, and o_j represent the actual network output. The descending technique is a common approach aimed at reducing the value of a function.

(2) Principal component Analysis Networks (PCA)

The PCA is a dimensionality reduction technique that can change m-dimensional to p-dimensional space. The calculation of PCA uses a recursive formulation. The PCA takes on the role of data compression and feature extraction.

The PCA algorithm is designed to normalize a distribution to ensure that there is no mean or unity variance, and it is calculated as follows:

$$A = \sum_P \left(X_p - \bar{X}\right)^T \left(X_p - \bar{X}\right), \tag{5}$$

where X_p is the training sample denoted by p.

The calculation of this study arranges the eigenvectors and their corresponding eigenvalues in descending order. Wen et al. (2020) compared the traditional stock price prediction models and found that the PCA model had better prediction performance with less value of the RMSE and MAPE.

(3) Radial Basis Function (RBF)

The RBF network is a hybrid learning process that uses Gaussian basis functions $\varphi_i(x)$. The initial step involves a supervised learning process, while the subsequent step utilizes an unsupervised learning process.

$$y = F(x) = \sum_{i=1}^{k} w_i \ \varphi_i(x), \ F(x) = \sum w_i \, G(\|x - t_i\|). \tag{6}$$

Sohrabi et al. (2023) calculated the coal prices by combining two-time series and combined radial basis function (RBF) neural network methods. The RBF has produced a more accurate prediction compared to the time series method.

3.2.4 Recurrent Neural Networks (RNN)

RNN is a dynamic neural network that applies time factors to the loop in the structure of the network based on three layers: input, hidden, and output. The structure of each output layer unit is connected to its structure through a feedback loop. The future stock market values were predicted using Recurrent Neural Networks (RNN) and a Long-Short Term Memory model (LSTM) by Hamiche and Moghar (2020).

From an input space to an internal state space calculated by RNN, which iterated algorithm to produce recurrent weight layer. The network is represented as:

$$y_i(t) = f\left(net_i(t)\right), \quad net_i(t) = \sum_i^n x_i(t) v_{ji} + \sum_h^m yh\left((t-1)u_{jh}\right) + \theta_j, \tag{7}$$

where *m* refers to the number of hidden nodes.

Determining state enables a set of output weights (w) to produce the network output.

$$y_k(t) = g\left(xdt_k(t)\right), \ net_i(t) = \sum_j^m y_j(t) w_{kj} + \theta_k. \tag{8}$$

where y_k refer to the output, w_{kj} stand for the weight associated with the output layer. MSE and RMSE statistical tests are used to evaluate the performance of the proposed ANN models. The forecasting precision performance is better when the value is smaller.

4. The Results

This research utilizes the GRA approach to rank input variables as key elements and uses the ANN approach to increase the accuracy of the estimation.

4.1 Empirical Results of GRA

The GRA used only one sequence x0(k) in this research. The results of GRA are shown in Table 3. The results showed that X4 (Industrial production index), X5 (Money Supply), X6 (Stock price index), X7 (Iron and Steel index), X8 (Hot-Rolled Coil Steel price), X9 (Brent crude oil price), X10 (BDI), X11 (CRB index) and X12 (PJM) are expected to have large measurements;[§] X1 (unemployment rate) and X2 (interest rate) are expected to have small measurements; and X3 (consumer price index) and X11 (CRB index) are expected to have nominal measurements.[**]

Table 3. Grey Relational Grades

Variables	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
EUA	13.571	13.993	13.877	13.789	14.000	3.763	13.853	12.954	14.133	11.766	6.715	14.105
Rank	8	4	5	7	3	12	6	9	1	10	11	2
GLCARB	8.516	8.590	8.547	8.520	8.577	2.615	8.534	8.149	8.615	7.766	3.827	8.618
Rank	8	3	5	7	4	12	6	9	2	10	11	1
KRBN	2.120	2.037	2.204	2.118	2.147	2.275	2.444	2.119	2.637	1.944	1.587	1.984
Rank	6	9	4	8	5	3	2	7	1	11	12	10

Note: X1= Unemployment rate; X2= Interest rate; X3= Consumer price index; X4= Industrial production index; X5=Money supply; X6= Stock price index; X7= Iron & Steel index; X8= Hot-Rolled Coil Steel price; X9= Brent oil price; X10= Baltic Dry Index; X11= CRB index; X12=PJM.

For the EUA, Brent oil price, PJM, Money Supply, interest rate, and consumer price index are the variables with the highest correlation, whereas stock price index, Baltic Dry Index, CRB index, Hot-Rolled Coil Steel price, Unemployment rate, and Industrial production index are the variables with the lowest correlation. The GLCARB index has the best correlation between the PJM, consumer price index, and money supply, the top three variables. For the KRBN, Brent oil price, Iron & Steel index, and Stock price index are the Top 3 variables with the highest correlation, resulting in economic variables that affected the EU ETS index. Chevallier (2011b) found that the stock market relationship is poorly predicted. Electric price is a significant factor in the highest sensitive indicator to the two carbon markets, as Huimin (2013) reported.

4.2 Empirical Results of ANN

This paper used ANN, including the BPN, PCA, RBF, and RNN models, to test the prediction results from the GRA model. Two statistical tests are carried out to evaluate the performance of the proposed ANN model. The network's errors can be reduced when MSE and RMSE are smaller. Hence, the producing value and the actual value of network estimation are consistent.

As shown in Table 4, this study found that RNN has better forecasting results than the others for the EUA. The BPN has better forecasting results than the others for the KRBN (MSE=0.0094) and GLCARB (MSE=0.0056). Carraro et al. (2009) revealed that the EUA

[§] "The larger, the better" or "The smaller, the better" is assumed that the independent variables were considered a large or small value, if both of the independent variables and the dependent variables were expressed as positively or negatively correlated, respectively.

[**] "Nominal is the best" is assumed that the independent variables were considered a small value if both the independent variables and the dependent variables were expressed as uncertainly correlated.

index has been influenced by policy and regulatory issues with higher uncertainty because of the different market bases on diverse driving factors.

Table 4. The Comparison of Forecasting Ability on Neural Network for Indexes

	Criteria	BPN	PCA	RBF	RNN
EUA	MSE	0.0084	0.0340	0.0324	**0.0072**
	RMSE	0.1707	0.6951	0.6623	**0.1535**
GLCARB	**Criteria**	**BPN**	**PCA**	**RBF**	**RNN**
	MSE	**0.0056**	0.0067	0.0102	0.0059
	RMSE	**0.0711**	0.0845	0.1281	0.0851
KRBN	**Criteria**	**BPN**	**PCA**	**RBF**	**RNN**
	MSE	**0.0094**	0.0115	0.0247	**0.0683**
	RMSE	**0.1364**	0.1662	0.3575	0.9875

The study splits the original data into two sub-samples training and remaining pattern testing datasets using the 10%, 20%, 33%, and 50%. The training data set was used as the primary estimate, while the test data set was used to evaluate the out-of-sample results. As shown in Table 5, the four ANN models indicated that the 10% testing sample has the minimum KRBN error, and the RBF network has better forecasting results than the others (MSE=0.0065). The 33% testing sample has the lowest error for the GLCARB index, and RNN networks have superior forecasting results than the others (MSE=0.0053). BPN, PCA, RBF, and RNN networks demonstrated that the 50% testing sample favors the EUA. The empirical results also showed that the RNN model exceeded the forecasts for the EUA.

4.3 Testing of GRA Results

GRA is applied to determine variables with the greatest impact on carbon prices, and ANN provides additional empirical tests of the power of their predictability. Table 6 displays the BPN network's result when all variables are used. Ranking performance is a way to rank performance by grouping the first six and last six variables. It is better to have the first six variables than the last six variables. The results indicated that the first six variables could predict carbon price.

Table 5. The Comparison of Forecasting Ability on Neural Network with Testing Sample

Index	EU ETS				GLCARB				KRBN			
	MSE		RMSE		MSE		RMSE		MSE		RMSE	
BPN	10%	0.0157	10%	0.201	10%	0.0097	10%	0.1126	10%	0.0066	10%	0.3477
	20%	0.6312	20%	2.1642	20%	0.2731	20%	2.469	20%	0.0154	20%	0.1542
	33%	0.0541	33%	1.1268	33%	0.0054	33%	0.0706	33%	0.0074	33%	0.1281
	50%	0.0096	50%	0.198	50%	2.6231	50%	3.0492	50%	0.0361	50%	1.2852
	Average	0.17765	Average	0.9225	Average	0.7278	Average	1.4254	Average	0.0164	Average	0.4788
PCA	10%	0.027	10%	0.3463	10%	0.0133	10%	0.1545	10%	0.0082	10%	0.4313
	20%	0.639	20%	2.191	20%	0.275	20%	2.4858	20%	0.0213	20%	0.2131
	33%	0.0559	33%	1.1644	33%	0.0078	33%	0.1019	33%	0.0085	33%	0.1464
	50%	0.0096	50%	0.194	50%	2.3758	50%	2.7618	50%	0.0415	50%	1.4791
	Average	0.182875	Average	0.973925	Average	0.6680	Average	1.3760	Average	0.0199	Average	0.5675
RBN	10%	0.0255	10%	0.3269	10%	0.0165	10%	0.1912	10%	0.0065	10%	0.343
	20%	0.5905	20%	2.0247	20%	0.2335	20%	2.1111	20%	0.032	20%	0.3195
	33%	0.0442	33%	0.9199	33%	0.0123	33%	0.1604	33%	0.018	33%	0.3094
	50%	0.0093	50%	0.1897	50%	3.0976	50%	3.6008	50%	0.0398	50%	1.4162
	Average	0.1674	Average	0.8653	Average	0.8400	Average	1.5159	Average	0.0241	Average	0.5970
RNN	10%	0.0369	10%	0.4447	10%	0.0086	10%	0.0999	10%	0.0157	10%	0.8292
	20%	0.0107	20%	0.1672	20%	0.2664	20%	2.4078	20%	0.1526	20%	1.5221
	33%	0.0795	33%	1.3464	33%	0.0051	33%	0.0671	33%	0.0167	33%	0.2863
	50%	0.0062	50%	0.1261	50%	2.9543	50%	3.4343	50%	0.1116	50%	1.1248
	Average	0.0333	Average	0.5211	Average	0.8086	Average	1.5023	Average	0.0742	Average	0.9406

Table 6. Substitute Volatility Indexes into Grey Relational Analysis for Neural Network

Index	Criteria	EUA	GLCARB	KRBN
All variables	MSE	0.0084	0.0056	0.0094
	RMSE	0.1707	0.0711	0.1364
The first six variables	MSE	0.0167	0.0051	0.0102
	RMSE	0.3100	0.0649	0.1378
The last six variables	MSE	0.0217	0.0116	0.0135
	RMSE	0.4024	0.1482	0.1829
Whether the first six variables are better than the last six variables?		YES	YES	YES

5. Conclusions

This research used four ANN models to forecast carbon prices, and twelve economic variables were ranked using GRA. The characteristics of a carbon price are established to enhance the prediction ability of global carbon price fluctuations. ANN models can be used as part of the GRA model to examine the best combination of variables for carbon price prediction. The results of the MSE and RMSE tests showed that the RNN network is smaller than other prediction models. The RNN was found to be more suitable for the EUA carbon price forecast by this paper.

This study found that using the first six variables is the most effective way to achieve optimal prediction accuracy, compared to using the last six variables and all variable groups. An extensive set of key variables must be considered to identify a consistent carbon price scenario. The conclusion also provides policy implications. As confirmed by previous studies, ANN applications have been shown to enhance forecasting ability. Using the GRA model to simplify the ANN model can assist investors in simplifying their carbon indices forecast by screening more influential variables.

This study found that the correct selection of training algorithm was necessary to maximize the predictive ability. Therefore, the testing sample has the average minimum error for the KRBN index, and the RBF network has better forecasting results than the others. Accurate carbon pricing forecasts inform carbon market policymakers and investors.

Reference

Alberola, E., Chevallier J, and Chèze B. (2008), "Price drivers and structural breaks in European carbon price 2005–2007", *Energy Policy*, 36(2), 787–97.

Benz, E. and Truck, S. (2009), "Modeling the price dynamics of CO2 emission allowances", *Energy Economics*, 31 (1), 4–15.

Carraro, Carlo and Alice, Favero (2009), "The economic and financial determinants of carbon prices", *Czech Journal of Economics and Finance*, 59, 396-409.

Cen, Z. and Wang, J. (2019), "Crude oil price prediction model with long short term memory deep learning based on prior knowledge data transfer", *Energy*, 169, 160-171.

Chang, F. J. and Chang, L. C. (2005), "Artificial neural networks induction", *Tsanghai book publishing company*.

Charkha, P. R. (2008), "Stock price prediction and trend prediction using neural networks", *In 2008 First International Conference on Emerging Trends in Engineering and Technology*, 592-594.

Chevallier, J. (2011a), "Evaluating the carbon-macroeconomy relationship: Evidence from threshold vector error-correction and Markov-switching VAR models", *Economic Modelling*, 28(6), 2634-2656.

Chevallier, J. (2011b), "Macroeconomics, finance, commodities: Interactions with carbon markets in a data-rich model", *Economic Modelling*, 28(1–2), 557-567.

Chevallier, J. (2011c), "A model of carbon price interactions with macroeconomic and energy dynamics", *Energy Economics*, 33(6), 1295-1312.

Chevallier, J. (2009d), "Carbon futures and macroeconomic risk factors: A view from the EU ETS", *Energy Economics*, 31(4), 614-625.

Demailly, D. and Quirion, P. (2008), "European emission trading scheme and competitiveness: A case study on the iron and steel industry", *Energy economics*, 30(4), 2009-2027.

Deng, J. L. (1982), "Control problems of grey systems", *Systems Control Letter 5*, 288-294.

Deng, J. L. (1989), "Introduction to grey system theory", *Journal of Grey System*, 1(1), 1-24.

Fæhn, T., Gómez-Plana, A. G., and Kverndokk, S. (2009), "Can a carbon permit system reduce Spanish unemployment?", *Energy Economics*, 31(4), 595-604.

Feng, C. M. and Wang, R. T. (2000), "Performance evaluation for airlines including the consideration of financial ratios", *Journal of Transport Management*, 6, 133-142.

Frunza, M. C. S. France and D. Guégan (2010), "Forecasting strategies for carbon allowances prices: From classic arbitrage pricing theory to switching regimes", *International Review of Applied Financial Issues and Economics*, 2(3), 576-596.

Hamiche, M., Moghar, A. (2020), "Stock market prediction using LSTM recurrent neural network", *Procedia Computer Science*, 170,1168-1173.

Hamzaçebi, C. and M. Pekkaya (2011), "Determining of stock investments with grey relational analysis", *Expert Systems with Applications*, 38(8), 9186-9195.

Hintermann, B. (2010), "Allowance price drivers in the first phase of the EU ETS", *Journal of Environmental Economics and Management*, 59(1), 43-56.

Huang, S.-M., Tsai, C.-F., Yen, D.C., and Cheng, Y.-L. (2008), "A hybrid financial analysis model for business failure prediction", *Expert Systems with Applications*, 35(3), 1034-1040.

Huimin, L. (2013), "The impact of human behavior on ecological threshold: Positive or negative?—Grey relational analysis of ecological footprint, energy consumption and environmental protection", *Energy Policy*, 56(0), 711-719.

Kanen, J. L. M. (2006), "Carbon trading and pricing", *Environmental Finance Publications*.

Kim, S. (1998), "Time-delay recurrent neural network for temporal correlations and prediction", *Neurocomputing*, 20(1), 253-263.

Kung, C.-Y. and K.-L. Wen (2007), "Applying grey relational analysis and grey decision-making to evaluate the relationship between company attributes and its financial performance—A case study of venture capital enterprises in Taiwan", *Decision Support Systems*, 43(3), 842-852.

Lin, F. and N. C. S. Sim (2013), “Trade, income and the Baltic Dry Index”, *European Economic Review*, 59, 1-18.

Liu, Lan-Cui, Ying, Fan, Gang, Wu, and Wei, Yi-Ming. (2007), “Using LMDI method to analyze the change of China's industrial CO2 emissions from final fuel use: An empirical analysis”, *Energy Policy*, 35(11), 5892-5900.

Mansanet, Bataller M., Pardo, A., and Valor, E., (2007), “CO2 prices, energy and weather”, *The Energy Journal*, 28(3), 67–86.

Ma, W., Wang, Y., and Dong, N. (2010), “Study on stock price prediction based on BP neural networ”, In 2010 IEEE International Conference on Emergency Management and Management Sciences , 57-60.

Movagharnejad, K., Mehdizadeh, B., Banihashemi, M., and Kordkheili, M. S. (2011), “Forecasting the differences between various commercial oil prices in the Persian Gulf region by neural network”, Energy, 36(7), 3979-3984.

Nelson, D. M., Pereira, A. C., and De Oliveira, R. A. (2017). “Stock market's price movement prediction with LSTM neural networks”, *International joint conference on neural networks (IJCNN)* , 1419-1426.

Niu, S., Ding, Y. Niu, Y. Li, Y., and Luo, G. (2011), “Economic growth, energy conservation and emissions reduction: A comparative analysis based on panel data for 8 Asian-Pacific countries”, *Energy Policy*, 39(4), 2121-2131.

Oberndorfer, U. (2009), “EU Emission Allowances and the stock market: Evidence from the electricity industry”, *Ecological Economics*, 68(4), 1116-1126.

Rahman, A., Srikumar, V., and Smith, A. D. (2018), “Predicting electricity consumption for commercial and residential buildings using deep recurrent neural networks”, *Applied Energy*, 212, 372-385.

Sohrabi, P., Jodeiri Shokri, B., and Dehghani, H. (2023) "Predicting coal price using time series methods and combination of radial basis function (RBF) neural network with time series", *Mineral Economics*, 36, 207–216

Thema, J., Suerkemper, F., Grave, K., and Amelung, A. (2013), “The impact of electricity demand reduction policies on the EU-ETS: Modelling electricity and carbon prices and the effect on industrial competitiveness”, *Energy Policy*, 60, 656-666.

Tucker, M. (1995), “Carbon dioxide emissions and global GDP”, *Ecological Economics*, 15(3), 215-223.

Wang, H. Z., Li, G. Q., Wang, G. B., Peng, J. C., Jiang, H., and Liu, Y. T. (2017). “Deep learning based ensemble approach for probabilistic wind power forecasting”, *Applied energy*, 188, 56-70.

Wen, Y. L., Lin, P.G., and Nie, X. S. (2020), “Research of stock price prediction based on PCA-LSTM model. ” *Materials Science and Engineering*, 790(1). 12109

Zagaglia, P. (2010), “Macroeconomic factors and oil futures prices: A data-rich model”, *Energy Economics*, 32(2), 409-417.

Zhang, J. S. and Xiao, X. C. (2000), “Predicting chaotic time-series using recurrent neural network”, *Chinese Physics Letter*,17(2), 88-90.

Zhang, F. and Wen, N. (2022), “Carbon price forecasting: A novel deep learning approach”, Environmental Science and Pollution Research, 29(36), 54782-54795.

Zhu, B., and Wei, Y. (2013), “Carbon price forecasting with a novel hybrid ARIMA and least squares support vector machines methodology”, Omega, 41(3), 517-524.